UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 3RD FEBRUARY 2000

Organised by the **United Kingdom Mathematics Trust**
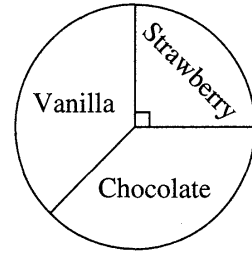**from the School of Mathematics, University of Leeds**

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.

2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.

3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.

4. Candidates in England and Wales must be in School Year 11 or below.
   Candidates in Scotland must be in S4 or below.
   Candidates in Northern Ireland must be in School Year 12 or below.

5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the
   Answer Sheet for each question. Do not mark more than one option.

6. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15.
   When you have checked your answers to these, have a go at some of the later questions.

7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.

8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the
   sheet except to mark your chosen options**. The machine 'sees' all black pencil markings
   even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of
   rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.

9. The questions on this paper challenge you to **think**, not to guess. You get more marks,
   and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK IMC is about solving interesting problems, not about lucky guessing.

1. 567 is multiplied by 3489. What is the units digit of the answer?

   A 1   B 3   C 5   D 7   E 9

2. An ice cream stall sells vanilla, strawberry and chocolate ice creams. The pie chart illustrates the sales of ice cream for last Saturday. The number of vanilla and the number of chocolate ice creams sold were the same. The stall sold 60 strawberry ice creams. How many chocolate ice creams were sold?

   A 90   B 99   C 100   D 120   E 135

3. Which is the largest of these fractions?

   A $\dfrac{7}{15}$ B $\dfrac{3}{7}$ C $\dfrac{11}{23}$ D $\dfrac{4}{9}$ E $\dfrac{6}{11}$

4. In Worcestershire, Wyre Piddle is 12km south of the village of North Piddle and Abbotts Morton is 12km east of North Piddle. What is the direction of Abbotts Morton from Wyre Piddle?

   A South East   B South West   C North East   D North West   E West

5. In a magic square, each row, each column and both main diagonals have the same total. In the partially completed magic square shown, what number should replace x?

   A 15   B 18   C 21   D 24   E 27

18			
13	15		
	10	11	17
	x	16	14

6. Granny has been having a smashing time. Yesterday she had 12 cups and 10 matching saucers, but this morning she dropped a tray holding one third of the cups and half the saucers, breaking all of those on the tray. How many of her cups are now without saucers?
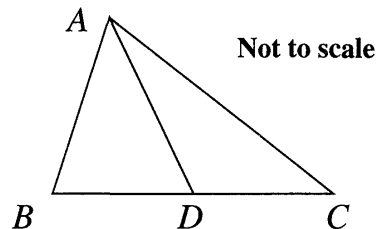
   A 1   B 3   C 4   D 5   E 6

7. Given that x and y are positive whole numbers and $x^2 + 2 = y^3$, which of the following is a possible value of x?

   A 2   B 3   C 4   D 5   E 6

8. In the triangle ABC, $AD = BD = CD$. What is the size of angle BAC?

   A 60°   B 75°   C 90°   D 120°

   E more information is needed

   Not to scale

9. Leap years normally occur every four years. However, years at the turn of a century are leap years only if they are multiples of 400. Therefore this year, 2000, is a leap year, but the year 1900 was not a leap year. How many leap years will there be between 2001 and 3001?

   A 240   B 242   C 248   D 249   E 250

10. The average (mean) weight of five giant dates was 50g. Kate ate one and the average (mean) weight of the four remaining dates was 40g. What was the weight of the date that Kate ate?

    A 10 g   B 50 g   C 60 g   D 90 g   E more information is needed

11. My bargain settee cost me £240 in a sale offering 25% reductions on all items. How much did I save?
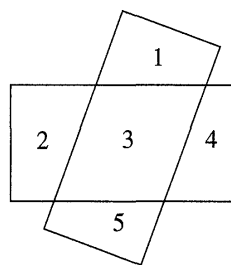
    A £25   B £40   C £60   D £80   E £100

12. Timmy, Tammy and Tommy all have tummy ache! They all set off separately to visit their doctor, leaving their homes at exactly the same time. Timmy cycles the 8 km to the surgery at an average speed of 20 km/hr; Tammy walks the 1.2 km to the surgery at an average speed of 4 km/hr and Tommy drives the 16.5 km to the surgery at an average speed of 45 km/hr. In what order do they arrive at the surgery?

    A Tommy, Timmy, Tammy   B Timmy, Tommy, Tammy   C Timmy, Tammy, Tommy

    D Tammy, Timmy, Tommy   E Tammy, Tommy, Timmy

13. The diagram shows two rectangles which enclose five regions. What is the largest number of regions which can be enclosed by any two rectangles drawn on a sheet of paper?

    A 10   B 9   C 8   D 7   E 6

14. The ratio $a : b = 2 : 3$ and the ratio $a : c = 3 : 4$. What is the ratio $b : c$?

    A 1 : 8   B 1 : 2   C 8 : 9   D 9 : 8   E 2 : 1

15. In how many whole numbers between 100 and 999 is the middle digit equal to the sum of the other two digits?

    A 28   B 36   C 45   D 50   E 55

16. The pattern 123451234512345... is continued to form a 2000-digit number. What is the sum of all 2000 digits?

    A 6000   B 7500   C 30 000   D 60 000   E 75 000

17. Baldrick can afford to buy either 6 parsnips and 7 turnips or else 8 parsnips and 4 turnips. Both options leave him with no change whatsoever. If, however, he bought only his beloved turnips, how many could he afford?

    A 11   B 12   C 13   D 16   E 25
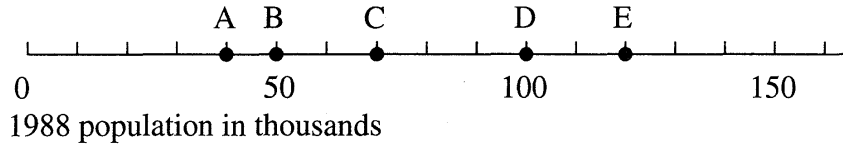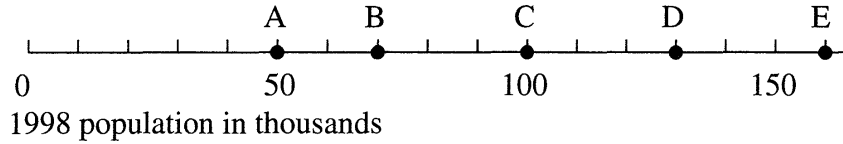
18. The number $3^4 \times 4^5 \times 5^6$ is written out in full. How many zeros are there at the end of the number?

    A none   B 4   C 5   D 6   E more than 6

19. The product of Mary's age in years on her last birthday and her age now in complete months is 1800. How old was Mary on her last birthday?

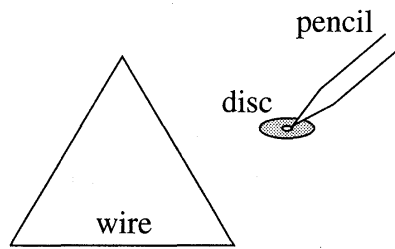   A 9   B 10   C 12   D 15   E 18

20. The populations of five cities A, B, C, D, E in 1988 and 1998 are shown on these scales.

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              A    B        C        D        E
    |_ _|_ _|_ _|_ _|_●_|_ _●_|_ _|_ _●_|_ _|_ _●_
    0         50        100       150
    1998 population in thousands


              A  B    C      D    E
    |_ _|_ _|_ _|_ _●_●_|_ _●_|_ _|_ _●_|_ _●_|_ _|_ _|_ _|
    0         50        100       150
    1988 population in thousands
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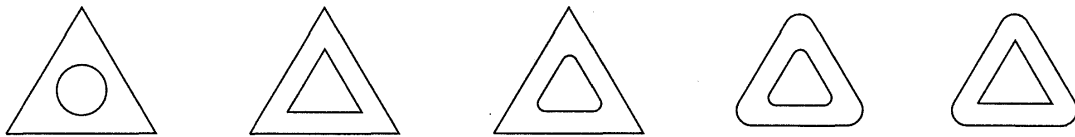
Which of the five cities showed the largest percentage increase in population from 1988 to 1998?

A   B   C   D   E

21. A wire in the shape of an equilateral triangle
    with sides of length 9 cm is placed flat on a piece
    of paper. A pencil is held in the hole at the
    centre of a disc of radius 1 cm, and the disc is
    rolled all the way around the outside of the wire,
    and then all the way around the inside of the
    wire. What shape is drawn by the pencil?

A   B   C   D   E

22. One hundred and twenty students take an exam which is marked out of 100 (with no
    fractional marks). No three students are awarded the same mark. What is the smallest
    possible number of pairs of students who are awarded the same mark?

    A 9   B 10   C 19   D 20   E 60

23. *ABCDEFGHI* is a regular nine-sided polygon (called a 'nonagon' or 'enneagon'). What is
    the size of angle *FAE*?

    A 10°   B 20°   C 30°   D 40°   E 50°

24. Jasmine spends exactly £120 on three types of plants: poisoned ivy, deadly nightshade and
    triffids. Poisoned ivy plants cost £2 each, deadly nightshade plants cost £9 each and
    triffids cost £12 each. She buys twenty plants in total, including at least one of each type.
    How many triffids did she buy?

    A 1   B 2   C 3   D 4   E more information is needed

25. The large circles in each figure have the same radius. Which shaded area is the greatest?

    A   B   C   D   E